

Mixed Boundary Conditions and Brane-String Bound States

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In this article we consider open strings with mixed boundary conditions (a combination of Neumann and Dirichlet conditions at each end). We discuss how their end points show a D_p -brane with NS-NS charge, i.e. **a bound state** of a D-brane with a fundamental strings. We show that these branes are BPS saturated. In the case of one-branes, we show that their mass densities are in agreement with IIB SUGRA which is $Sl(2, \mathbb{Z})$ invariant. Using Chan-Paton factors, we extend our results to the case of bound states of n D-strings and m F-strings. These string theoretic results are also checked in the effective field theory limit.

1 Introduction

Since their discovery by Polchinski [1,2], D-branes have contributed immensely to our understanding of various phenomena in superstring theories. By considering their interactions via the exchange of closed strings, one can recover their long range behaviour which get contributions from massless fields graviton, dilaton, and RR fields. One finds that they are BPS saturated objects with a unit of the proper RR charge and that their mass density is proportional to $\frac{1}{g_s}$ [1], as predicted by the effective field theories (SUGRA). It has also been shown that we can construct bound states of such objects with themselves [3,4,5] or with F(undamental)-strings [3]. These bound states may or may not be BPS states. The bound states can be constructed from branes of the same or different dimensions. Among these bound states of p - and p' -branes, those with $p' = p + 2$ are truly bound states[4], i.e, their energy is lower than the sum of the individual energies. For $p' = p + 4$ or $p' = p$, due to SUSY, they are marginally bound (their masses are exactly the sum of the masses of the individual branes). In the case of bound state of n similar D-branes, one can check that the corresponding brane system is BPS saturated carries n units of RR charge [3]. In this case, open strings attached to D-branes look like Chan-Paton gauged strings with gauge group $U(n)$.

Besides bound state of D-branes, we can have BPS bound states of F-strings with D-branes. Witten has shown that these states carry the charge of a $U(1)$ gauge field living in the D-brane [6,3], which is partly the pull back of the background Kalb-Ramond field. Since the $U(1)$ charge in D-string case is the momentum conjugate to $U(1)$ gauge field, the corresponding charge acquires integer values through quantization [3]. If we consider other D_p -branes, the electric $U(1)$ charge is again related to momentum and field strength of $U(1)$ field.

We show that the (F-string)-(D-brane) BPS *bound states* can be represented in string theory by *mixed boundary conditions* on open strings attached to branes. The mixed boundary conditions has been first considered in [6]. A particular class of mixed boundary conditions have been perviously considered [5,7] to describe bound states of p and $p-2$ -branes, which do not carry NS-NS charge [8]. They are arrived at by applying T-duality to a direction which is niether perpendicular nor parallel to a $D_{(p-1)}$ -brane. The boundary conditions are then those of a D_p -brane with an internal magnetic gauge field, i.e. the mixed boundary conditions do not involve the time. Although these works considered mixed boundary conditions, their calculations are all within the context of SUGRA and not within string theory. Mixed boundary conditions have also been encountered in [9], where again magnetic Wilson lines

are assumed to exist in the six dimensional compact manifold of type I ($N = 1$) theories.

In this article the mixed boundary conditions are extended to include branes with internal electric field. As a result the mixed boundary conditions also involve the time direction. This extension will have novel implications: the brane will become a bound state of a D-brane and a number of F-strings. In contrast to the magnetic case [5,7], the branes we consider carry charge of NS-NS Kalb-Ramond field. This fact can be observed in variety of ways. First, we can derive it applying a chain of T-dualities, discussed in section 3. Second, considering the DBI action and the B field equation of motion, we find that F_{0i} is proportional to the NS-NS charge density of bound state. Third, within string theory, finding the long range interaction of such bound states, we show the existence of the $B_{\mu\nu}$ charge. The third argument constitutes a generalization of the Polchinski's work to (F-string)-(D-brane) bound states.

In general, we will use the $(m, 1_p)$ notation for bound states of D_p -branes and F-strings, in which m determines the magnitude of NS-NS two form charge ¹, and 1_p determines charge of the proper RR form (($p+1$) RR form).

As discussed in the sequel, the mixed boundary method is capable of representing the NS-NS charge ², i.e., they enable us to construct a string theoretic description of these bound states.

Here we mainly focus on one-branes in IIB theory. These objects are needed for $Sl(2,Z)$ duality of IIB strings [10]. Using the conventional notation we denote their charges (i.e., NS-NS, Kalb-Ramond, and RR two form, respectively) charges by (m, n) , an $Sl(2,Z)$ doublet.

In section 2, we review the derivation of mixed boundary conditions from a σ -model action [6] and by means of open strings stretched between two $(m, 1)$ strings. Using the standard techniques of [1], we calculate their interactions. Their vanishing indicates that the branes defined by mixed boundary conditions are BPS saturated and preserve one-half of the SUSY, like $(0, 1)$ strings introduced by Polchinski [1]. We also obtain their mass density which again is a check for BPS conditions. The problem of these bound states in the case of P -branes ($(m, 1_p)$ branes, $p > 1$) is also discussed.

In section 3, we will analyze how NS-NS charged branes of IIB theory are related to moving D -branes in IIA string theory under T-duality. Our discussion sheds light on the problem of the relation between $Sl(2,Z)$ symmetry of IIB string theory and T-duality of IIA and IIB theories [11].

¹The $B_{\mu\nu}$ charge is a two form defined in the world-volume of brane.

² Unless it is mentioned explicitly, by NS-NS charge we mean the NS-NS two form charge.

In section 4, we discuss how $(m, 1_p)$ bound states, similar to $(0, 1_p)$ states, show symmetry enhancement when they are coincident. Moreover, we explain how these symmetry enhancements in the case of IIB D-strings or D_5 -branes also hold in the strong coupling regime. Equivalently, we will present a $Sl(2, Z)$ invariant argument for symmetry enhancement.

In section 5, we show how (m, n_p) branes can be described by means of both Chan-Paton gauged open strings and by mixed boundary conditions. Calculating the interactions of two (m, n_p) branes, we obtain their mass density and discuss their symmetry enhancements in the coincident limit.

In section 6, we present the field theoretic description of what we have done in previous sections. We do this using the generalized ($Sl(2, Z)$ invariant) DBI-type action [12] and IIB SUGRA [5]. Again we calculate the (m, n) string interactions at the tree level from field theory which is in agreement with the long range results of string theory.

In section 7, we will discuss some new features and open questions.

2 Mixed Boundary Conditions:

One can introduce Dirichlet boundary conditions on open strings by adding a constraint term to usual σ -model [6] as following:

$$S = \frac{1}{4\pi\alpha'} \int_{\Sigma} d^2\sigma [G_{\mu\nu} \partial_a X^\mu \partial_b X^\nu g^{ab} + \epsilon^{ab} B_{\mu\nu} \partial_a X^\mu \partial_b X^\nu + \alpha' \partial^a \rho \partial_a X^\mu \partial_\mu \Phi] + \frac{1}{2\pi\alpha'} \oint_{\partial\Sigma} d\tau A_i \partial_\tau \zeta^i, \quad (1)$$

where Σ and $\partial\Sigma$ are the world-sheet and its boundary, A_i $i = 0, 1, \dots, p$, the $U(1)$ gauge field living in a D_p -brane, ζ^i are its internal coordinates and G, B, Φ are usual back-ground fields. Variation of this action with respect to X^μ gives either of the following boundary conditions:

$$\left\{ \begin{array}{l} \delta X^\mu = 0 \\ G_{\mu\nu} \partial_\sigma X^\nu + F_{\mu\nu} \partial_\tau X^\nu = 0 \end{array} \right. \quad (2)$$

where

$$F_{\mu\nu} = \begin{cases} B_{\mu\nu} & \mu > p \\ B_{\mu\nu} - A_{[\mu,\nu]} & \mu, \nu \leq p. \end{cases} \quad (3)$$

As we see when there is a non-trivial $U(1)$ gauge field strength, $F_{\mu\nu}$, the usual Neumann boundary conditions is replaced by mixed boundary condition. So it suggests that we are able to introduce bound state of D-branes with F-strings (carrying a non-vanishing $B_{\mu\nu}$ field with a source on the brane and hence making a non-trivial $F_{0\nu}$ background) by mixed boundary condition on open strings attached to. A similar boundary conditions with non-zero $B_{\mu\nu}$ ($\mu, \nu \neq 0$) have been considered to discuss bound state of $p, p-2$ branes [5] in the context of SUGRA theories. It is worth noting that in those cases the $F_{\mu\nu}$ shows the distribution of $p-2$ on the p -brane world-volume. Similarly in our case ($F_{0i} \neq 0$), F_{0i} for the ($p > 1$) shows the NS-NS charge distribution. The $B_{\mu\nu}$ field found in [5] are source free unlike what we obtain here. In this article we mainly focus on D-string case.

For $(m, 1)$ string, following [3] we can argue that $F_{ab} = m\lambda\epsilon_{ab}$, $a, b = 0, 1$, where λ is the string coupling constant. So the related boundary conditions are:

$$\begin{cases} \delta X^\mu = 0 & \mu = 2, \dots, 9 \\ \partial_\sigma X^a + m\lambda\epsilon_{ab}\partial_\tau X^b = 0. \end{cases} \quad (4)$$

the corresponding fermionic boundaries are

$$\begin{cases} (\psi^\mu - (\frac{im-1}{im+1})\tilde{\psi}^\mu)|B>=0 & \mu = 0, 1 \\ (\psi^\mu + \tilde{\psi}^\mu)|B>=0 & \mu = 2, \dots, 9. \end{cases} \quad (5)$$

The value of the quantum of charge can be determined from the calculation on the disk, or equivalently by use of a one-loop vacuum amplitude. To do so let us consider two parallel $(m, 1)$ strings at $X^\mu = 0$ and $X^\mu = Y^\mu$ $\mu = 2, \dots, 9$. The one loop vacuum graph of mixed open strings (open strings with mixed boundary conditions) is sum over the cylinders with ends on each D-brane. In the closed strings channel this is exchange of single closed string between them.

Before the mode expansion for the mixed open strings stretching between branes and build their first quantized components.

Imposing (4) on $X^\mu(\sigma, \tau)$ at $\sigma = 0, \pi$ we have:

$$\left\{ \begin{array}{l} X^0 = x^0 + N(p^0\tau - m\lambda p^1\sigma) + N' \sum_{n \neq 0} \frac{e^{-in\tau}}{n} (a_n^0 \cos n\sigma + im\lambda a_n^1 \sin n\sigma) \\ X^1 = x^1 + N(p^1\tau + m\lambda p^0\sigma) + N' \sum_{n \neq 0} \frac{e^{-in\tau}}{n} (a_n^1 \cos n\sigma - im\lambda a_n^0 \sin n\sigma) \\ X^\mu = Y^\mu \frac{\sigma}{\pi} + \sum_{n \neq 0} a_n^\mu \frac{e^{-in\tau}}{n} \sin n\sigma \quad \mu = 2, \dots, 9, \end{array} \right. \quad (6)$$

where N, N' are some normalization factors which are determined by considering the canonical commutation relations

$$[X^0, P^0] = [X^1, P^1] = i. \quad (7)$$

As it is seen from the action (1) the conjugate momentum of the X^0, X^1 are:

$$\left\{ \begin{array}{l} P^0 = \partial_\tau X^0 + F_{01} \partial_\sigma X^1 \\ P^1 = \partial_\tau X^1 - F_{10} \partial_\sigma X^0 \end{array} \right. \quad (8)$$

By a gauge transformation introduced in [3] which is only a function of $(m, 1)$ string coordinates, F_{ab} can be set to be $m\lambda\epsilon_{ab}\delta_a^\mu\delta_b^\nu$. It is worth to note that the corresponding gauge transformation do not change the boundary conditions which is only a function of gauge invariant F .

For general $F_{\mu\nu}$ the extra term in the canonical momentum densities looks like world sheet gauge potentials. For constant F it only leads to a change in the normalization factor N which is obtained to be:

$$N = \frac{1}{1 + m^2\lambda^2}, \quad N' = N^{1/2}. \quad (9)$$

They lead to the expected commutation relations:

$$[a_n^\mu, a_m^\nu] = \delta_{n+m} \delta^{\mu\nu} \quad \mu, \nu = 0, \dots, 9. \quad (10)$$

The amplitude for the exchange of closed string is:

$$A = \int \frac{dt}{2t} \sum_{i,p} e^{-2\pi\alpha' t \mathcal{H}}, \quad (11)$$

where i indicates the modes of the open string and p the momentum which has non-zero value in 0,1 components, and \mathcal{H} is the open string world-sheet Hamiltonian, which is obtained from action (1). Performing integration on momentum part and trace on oscillatory modes we get:

$$A = 2V_2(1 + m^2\lambda^2) \int \frac{dt}{2t} (8\pi^2\alpha't)^{-1} e^{-\frac{Y^2 t}{2\pi^2\alpha'}} (\mathbf{NS} - \mathbf{R}), \quad (12)$$

where \mathbf{NS} and \mathbf{R} are given by

$$\mathbf{NS} = \frac{1}{2}q^{-1} \prod \left(\frac{1 + q^{2n+1}}{1 - q^{2n}} \right)^8 - \frac{1}{2}q^{-1} \prod \left(\frac{1 + q^{2n-1}}{1 - q^{2n}} \right)^8. \quad (13)$$

$$\mathbf{R} = 8 \prod \left(\frac{1 + q^{2n}}{1 - q^{2n}} \right)^8. \quad (14)$$

The novel feature of this result is the multiplicative factor involving λ which modifies the tension. The Total interaction vanishes, which is a sign of SUSY in open string channel (the corresponding solutions preserve half of the SUSY). In the exchanged closed string point of view this is a sign of BPS condition for the branes [13]. We can extract graviton and dilaton and RR, Kalb-Ramond contributions. In order to see the effective low energy contributions (massless closed strings) we go to $t \rightarrow 0$ limit:

$$A = [\mathbf{1} - \mathbf{1}](1 + m^2\lambda^2)V_2(2\pi)(4\pi^2\alpha')^2 G_8(Y^2). \quad (15)$$

The $-\mathbf{1}$ term in brackets is due to graviton, dilaton and the $\mathbf{1}$ term is due to the contribution of the RR and NS-NS two forms respectively; i.e. the term proportional to $m^2\lambda^2$ is due to $B_{\mu\nu}$ and 1 due to RR two form.

Vanishing amplitude as we will see explicitly in field theoretical calculations shows the BPS condition saturation and reminds the "No Force Condition" between BPS states which is also a sign of SUSY.

From now on we use the $Sl(2, \mathbb{Z})$ doublets consisting of the RR and NS-NS two forms, $(\tilde{B}_{\mu\nu})$ and $B_{\mu\nu}$:

$$\mathcal{B}_{\mu\nu} = \begin{pmatrix} B_{\mu\nu} \\ \tilde{B}_{\mu\nu} \end{pmatrix}. \quad (16)$$

Comparing with field theoretical results, we show explicitly in section 6 that how from (15) we get the $(m, 1)$ string BPS mass density formula (when the RR scalar vanishes) to be:

$$T_{(m,1)} = (4\pi^2\alpha')^{1/2} \sqrt{\frac{1}{\lambda^2} + m^2}. \quad (17)$$

where $(4\pi^2\alpha')^{1/2}$ is the F-string tension. The above results can easily be generalized to some special P -brane cases which are described by only non-zero F_{0i} . In these cases by

a coordinate transformation in world volume we can obtain a F_{0i} with only two non-zero components, e.g. the F_{01} and F_{10} .

In general $B_{\mu\nu}$ charge is a two form defined on the D-brane world-volume which in the case of string is just proportional to ϵ_{ab} in two dimensional world-sheet.

In these special cases the calculations presented here for the $(m, 1)$ strings, is not much altered. For $(m, 1_p)$ branes only two components of attached open strings have mixed, $(p-1)$ of them Neumann and the transverse $(9-p)$ components by Dirichlet boundary conditions. It is crucial that the mixed boundary conditions is imposed on X^0 , and an arbitrary X^i . Thus introducing a non-zero F_{0i} field, breaks the $SO(p, 1) \times SO(9-p)$ lorentz symmetry to $SO(1, 1) \times SO(p-1) \times SO(9-p)$. This anisotropy gives rise to a new intrinsic form i.e, the volume form in the $(0, i)$ subspace. The bulk fields can also be along this new form, in contrast to D-branes where only the total volume form is consistent with the symmetries which allows only non-vanishing $(p+1)$ forms.

For the case of two *similar* $(m, 1_p)$ branes interaction, again we obtain the result for the interaction of two similar D_p brane multiplied by the factor $(1 + m^2\lambda^2)$. Again vanishing of the amplitude signals presence of super symmetry and BPS property of the $(m, 1_p)$ -brane bound states. More over the tension; the constant in front of the amplitude, is

$$T_{(m, 1_p)} = T_{(p)}^0 \sqrt{1 + \lambda^2 m^2} \quad (18)$$

where T_p^0 is tension of a D_p -brane.

The state $(m, 1_p)$ inspite of being anisotropic is uniform which can be seen from the amplitude being proportional to the Green's function in the transverse space and also from the boundary conditions. Therefore the strings are uniformly distributed on a P-brane world-volume with density proportional to m . This explains the finite contribution of F-strings to the tension of bound state. In the world-volume super symmetry point of view, this is the share of the NS-NS charge density to the tension of the BPS state.

The system of two $(m, 1_p)$ branes with non-parallel NS-NS charges; their interactions does not vanish any more (the graviton,dilaton contrubutions do not cancel RR and Kalb-Ramond's). Interaction of these branes is studied in field theory limit, which we will explain it in section 6.

3 T-duality of IIA , IIB and $(m, 1_p)$ Bound states:

It is well known that, e.g.[14], the type IIA theory on a circle is T-dual to IIB. The problem we would like to address in this section is the identification of the states corresponding to $(m, 1_p)$ branes after T-duality. Let us consider a D_{p-1} -brane defined by the following boundary conditions for the end of the strings attached to it [15]:

$$\left\{ \begin{array}{l} \delta X^\mu = 0 \quad \mu = p+1, \dots, 9 \\ \partial_\tau (X^p - \Omega_\alpha^p X^\alpha) = 0 \\ \partial_\sigma (X^\alpha + \Omega_p^\alpha X^p) = 0 \\ \partial_\sigma X^\mu = 0 \quad \mu = 0, \dots, p-1, \mu \neq \alpha. \end{array} \right. \quad (19)$$

If $\alpha = 0$, these boundary conditions show a moving D_{p-1} -brane in the P-direction with velocity Ω_{0p} , and if $\alpha \neq 0$ they represent a D_{p-1} -brane rotated in (α, p) plane with angle $\theta = tg^{-1}(\Omega_{\alpha p})$. Under T-duality in X^p direction, ∂_σ and ∂_τ acting on X^p , are interchanged. Hence (19) is replaced with:

$$\left\{ \begin{array}{l} \delta X^\mu = 0 \quad \mu = p+1, \dots, 9 \\ \partial_\sigma X^p - \Omega_\alpha^p \partial_\tau X^\alpha = 0 \\ \partial_\sigma X^\alpha + \Omega_p^\alpha \partial_\tau X^p = 0 \\ \partial_\sigma X^\mu = 0 \quad \mu = 0, \dots, p-1, \mu \neq \alpha. \end{array} \right. \quad (20)$$

describing a $(m, 1_p)$ brane for the $\alpha = 0$ with

$$\Omega_{0p} = F_{0p}, \quad (21)$$

and for $\alpha \neq 0$ a $(p, p-2)$ brane bound state [5] in which $p-2$ branes are homogeneously distributed on P -brane world volume in (α, p) plane with:

$$\Omega_{\alpha p} = F_{\alpha p}. \quad (22)$$

It can be seen that in the case of our interest, the internal electric gauge field (F_{0i}), has acquired non-zero value under T-duality in the velocity direction. This means that in T-dualizing a type II theory(A or B), a $(p-1)$ -brane with momentum³, m/R is transformed

³R is the radius of compactification in the X^p direction.

to a bound state of a D_p -brane and m F-strings. The NS-NS charge density of this bound state is $m\lambda$. λ appears because of the relation between velocity and momentum, involving $(p-1)$ -brane mass density: $\frac{\alpha'^{p/2}}{\lambda}$.

A particular result of the above is that, a dual state corresponding to NS-NS charged strings is the momentum modes of the D_0 -branes.

We have shown that one can get $(m, 1_{p+1})$ brane of IIB (IIA) theory by T-dualizing moving D_p -branes of type IIA (IIB). Therefore $(m, 1)$ strings with different NS-NS charges are related by a boost transformation in the T-dual theory.

$$\begin{array}{ccc}
(m, 1)string & & (0, 1)string \\
| & & | \\
\text{T-duality } \uparrow & & \uparrow \text{ T-duality} \\
| & & | \\
(v = m\lambda) D_0 - brane & \xrightarrow{BOOST} & (v = 0) D_0 - brane
\end{array}$$

In the above diagram T-duality is done in boost direction.

It may seem that a $Sl(2, Z)$ transformation on type IIB [10] may project $(m, 1)$ state to $(0, 1)$ and hence complete the above diagram. Specifically the T subgroup of $Sl(2, Z)$ maps $(0, 1)$ to $(m, 1)$ but the obstacle is that such transformations also moves the type IIB theory in its moduli space and transforms the RR scalar from zero to m . On the other hand T-duality on type IIA theory takes it to a type IIB thoery with zero χ (the RR scalar). To go beynd $\chi = 0$ we need to consider the M-theory compactified on T^2 instead of $\frac{IIA}{S^1} ((M/S^1)/S^1)$.

4 Bound State of $(m, 1_p)$ branes and Symmetry

Enhancement:

Similar to the coincidence limit of D_p -branes [3], corresponding limit of $(m, 1_p)$ branes also shows symmetry enhancement. When n $(m, 1_p)$ branes coincide their internal field theory becomes a $U(n)$ SYM theory in $(1+1)$ dimensions.

There are n massless string states with their two ends on each of them. Between any two branes, i and j , two oriented strings can be suspended with mass proportional to

$|Y_i - Y_j|\sqrt{1 + m^2\lambda^2}$ which vanishes in the desired limit; $Y_i = 0 \ i = 0, \dots, n$. In this limit we have a total number of n^2 massless states forming the gauge vector for a $U(n)$ symmetry. To clarify the above argument for $(m, 1)$ string let us study the internal field theory of it.

In the case of D_p -brane or $(0, 1_p)$, the gauge field strength is partly pull back of $B_{\mu\nu}$:

$$F_{ab} = B_{ab} - A_{[a,b]}. \quad (23)$$

In the case of $(m, 1)$ string of type IIB $Sl(2, Z)$ duality requires the existence of an extra \tilde{A}_a field which is the pull back of $\tilde{B}_{\mu\nu}$, the RR two form [12]. The field strength of it is given by

$$\tilde{F}_{ab} = \tilde{B}_{ab} - \tilde{A}_{[a,b]}. \quad (24)$$

It is clear that F and \tilde{F} form a $Sl(2, Z)$ doublet

$$\mathcal{F}_{\mu\nu} = \begin{pmatrix} F_{\mu\nu} \\ \tilde{F}_{\mu\nu} \end{pmatrix}.$$

It may seem that we have two $U(1)$ gauge fields living in the $(m, 1)$ string world-sheet. This is not the whole story; a particular combination of the two gauge fields F and \tilde{F} decouples from the $(m, 1)$ string in the classical level. This leaves us with only one $U(1)$ gauge field in the $(m, 1)$ string world-sheet. To find this particular combination we use the $Sl(2, Z)$ invariant action for (m, n) strings [11]. In this action which we return to it more specifically in section 6, the classical solution of equation of motion contains fluctuations of \mathcal{F} which is perpendicular to state of $(m, 1)$ string in the moduli space. So among different \mathcal{F} only the combination which satisfies this condition remains:

$$(m, 1)\mathcal{M}\mathcal{F} = 0, \quad (25)$$

where \mathcal{M} is the moduli space metric (33). This equation is manifestly $Sl(2, Z)$ invariant, hence we can find the particular $U(1)$ gauge field from it:

$$\mathcal{F}_{\mu\nu} = \begin{pmatrix} F_{\mu\nu} \\ -m\lambda^2 F_{\mu\nu} \end{pmatrix}. \quad (26)$$

So in the coincidence limit, i.e. when n $(m, 1)$ strings are on top of each other they make a (nm, n) string in which, there is a $U(n)$ gauge theory.

It is necessary to bear in mind that symmetry enhancement of coincident similar D_p -branes for even p is only valid for the usual string theory limits ($g_s \ll 1$), at strong coupling

for IIA cases (even p) symmetry enhancement arguments fails [16]. In the case of type IIB theory symmetry enhancement argument is supported by $Sl(2,Z)$ duality and holds even in strong coupling.

5 (m, n_p) brane bound states in string theory

In this section we construct bound state of m F- strings with n D_p -branes in string theory. For this purpose we first consider n similar D_p -branes bound state, and then by imposing mixed boundary conditions on two components (X^0 and X^i) of open strings attached to brane, we obtain n D_p -brane m F-string bound state. By means of this definition, we study (m, n) strings interactions in string theory limit ($g_s \ll 1$, $\chi=0$). As a result we find their mass or charge density saturating the BPS condition, in agreement with SUGRA [10].

The open strings attached to n D_p -brane system form an adjoint representation of $U(n)$ gauge field [3].

The group theoretic state of these open strings is easily introduced by usual Chan-Paton factors given to each end. These Chan-Paton factors are in fundamental representation of $U(n)$ (quark and anti-quark), so that the whole open string sits in adjoint representation of $U(n)$. Explicitly one can write boundary condition of such open strings as:

$$\begin{cases} X^\mu = 0 & \mu = p+1, \dots, 9 \\ \partial_\sigma X^\mu = 0 & \mu = 0, \dots, p \end{cases} \quad (27)$$

where X^μ is an $n \times n$ $U(n)$ matrix. At each end ($\sigma = 0, \pi$) we represent its $U(n)$ state by $(\lambda_i, \bar{\lambda}_j)$ or $(\bar{\lambda}_i, \lambda_j)$ which $\lambda, \bar{\lambda}$ show the quark anti-quark representations ⁴.

Following above in presence of two n and n' D_p -branes the X^μ become $U(n+n')$ matrices. The $U(n+n')$ matrix can be divided in an obvious manner to four parts; an upper corner $U(n)$ part representing the states of strings ending at both ends on the n D_p -brane, the lower corner $U(n')$ which represents the state of strings at both ends attached to the n' D_p -brane and two $n \times n'$ and $n' \times n$ matrices for strings stretched between the two branes. So in the calculation of the interaction between n, n' parallel D_p -branes the sum on all the open string states includes a trace over the group theoretic states of the strings stretched between branes which are in the $U(n) \times U(n')$ representation of $U(n+n')$. The amplitude is simply

⁴these two distinct states show that these open strings are oriented strings.

the same amplitude for interaction of two unit RR charged D-branes times nn' . Vanishing of the amplitude shows that n D_p -branes on top of each other form a marginal BPS saturated state with mass proportional to $\frac{n}{g_s}$.

More generally we can extend the mixed boundary conditions to this group theoretic states. In this way we are able to construct bound state of m F- strings with any number of D_p -branes.

Here we restrict ourselves to (m, n) strings whose boundary conditions are:

$$\begin{cases} \delta X^\mu = 0 & \mu = 2, \dots, 9 \\ n\partial_\sigma X^a + m\lambda\epsilon_{ab}\partial_\tau X^b = 0 \end{cases} \quad (28)$$

and X^μ $\mu = 0, \dots, 9$ in adjoint representation of $U(n)$. So their state at $\sigma = 0, \pi$ are given by λ_i or $\bar{\lambda}_j$.

By this method we can calculate the (m, n) string interactions through the one-loop vacuum amplitude of these mixed Chan-Paton open strings:

$$A = n^2 \left(1 + \frac{m^2 \lambda^2}{n^2}\right) \times A_0, \quad (29)$$

where A_0 is the corresponding amplitude for two $(0, 1)$ strings. Thus at massless closed strings or effective field theory limit one can write (29) as:

$$A = (4\pi^2 \alpha') \left(\frac{n^2}{\lambda^2} + m^2\right) V_2(1-1) G_8(Y^2). \quad (30)$$

The above result shows that (m, n) string (like $(m, 1)$ strings) form a BPS bound state with mass density $(4\pi^2 \alpha')^{1/2} \sqrt{m^2 + n^2/\lambda^2}$. This is what one would expect from $Sl(2, Z)$ symmetry. We note that in general two (m, n) and (m', n') string case, the relative interactions do not cancel (except when $\frac{m}{n} = \frac{m'}{n'}$).

The gauge symmetry remaining after the formation of bound state (m, n) string (considering mixed boundary conditions) is $U(r)$ where r is the greatest common divisor of m and n . This is easily seen from a $Sl(2, Z)$ transformation which takes the (m, n) to $(0, r)$.

Although the above arguments are given for strings, we can generalize it to any (m, n_p) brane (bound state of n D_p -branes with m F-strings). In this case the mass density of such bound state is obtained to be (18) multiplied by n^2 due to the trace on group theoretic states. This shows that (m, n_p) branes are BPS saturated bound states of D_p -branes and F-strings. These bound state are related to (m, n) strings by a chain of T-dualities.

In the case of IIB (m, n) strings there is a $\text{Sl}(2, \mathbb{Z})$ invariant combination of \mathcal{F} components which is the surviving gauge field, like equation (25). The corresponding combination is determined from orthogonality condition:

$$(m, n) \mathcal{M} \begin{pmatrix} F \\ \tilde{F} \end{pmatrix} = 0 \quad (31)$$

More precisely every string state in moduli space of IIB theory is given by a (m, n) $\text{Sl}(2, \mathbb{Z})$ doublet. The transverse oscillations of these strings in the moduli space are described by $U(r)$ gauge fields related to massless states of attached open strings.

By a discussion similar to section 3, we can see that T-duality, in string spatial direction, transforms a (m, n) string to a bound state of n D_0 -branes moving with velocity $m\lambda$ in the compact direction.

6 Field Theory Descriptions, Manifest $\text{Sl}(2, \mathbb{Z})$:

In the previous sections we built the string theoretic description of bound states of D-branes with D-branes or F-strings. We can check our results in the IR limit with SUGRA results. In this section we restrict ourselves to one-branes of IIB theory. The field theory action consists of two parts; the IIB SUGRA action [5] and a generalized $\text{Sl}(2, \mathbb{Z})$ invariant DBI-type action [12] including an interaction term. The first describes the dynamics of the bulk NS-NS and RR two form fields, and the latter the dynamics of the (m, n) strings and their interactions with the bulk.

IIB SUGRA action

In order to build a IIB action, we use T-duality between IIA and IIB theories on an arbitrary direction, doing so we get to the following action [5]:

$$S_{IIB} = \frac{1}{2k^2} \int d^{10}x \sqrt{-g} [R + \frac{1}{4} \text{tr}(\partial \mathcal{M} \partial \mathcal{M}^{-1}) - \frac{1}{12} \mathcal{H}^t \mathcal{M} \mathcal{H} - \frac{1}{480} (dA^{(4)} - \mathcal{B}^t S \mathcal{H})^2] + \quad (32)$$

$$\frac{1}{4k^2} \int A^{(4)} \wedge \mathcal{H}^t S \mathcal{H}.$$

where we have used the notation of [10]: \mathcal{M} , a 2×2 $\text{Sl}(2, \mathbb{R})$ matrix is the metric in the

moduli space

$$\mathcal{M} = e^\phi \begin{pmatrix} |\lambda|^2 & \chi \\ \chi & 1 \end{pmatrix}. \quad (33)$$

with $\lambda = \chi + ie^{-\phi}$ (χ and ϕ are RR and NS-NS scalars respectively). \mathcal{B}_{ab} is the $\text{Sl}(2, \mathbb{R})$ doublet (16) and $\mathcal{H} = d\mathcal{B}$. $A^{(4)}$ is the usual self dual 4-form of IIB theory which is a $\text{Sl}(2, \mathbb{R})$ singlet. S is a constant 2×2 $\text{Sl}(2, \mathbb{R})$ matrix:

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (34)$$

Although we are concerned with self dual field $A^{(4)}$ [11], one can impose self duality conditions on by hand ⁵.

The manifest $\text{Sl}(2, \mathbb{R})$ invariance of (32), is broken to $\text{Sl}(2, \mathbb{Z})$ by quantum considerations.

Manifestly $\text{Sl}(2, \mathbb{Z})$ Invariant DBI-type Action for (m, n) Strings

As recently has been discussed by Townsend [12], there are two $\text{U}(1)$ induced gauge fields related to $\mathcal{B}_{\mu\nu}$, which form an $\text{Sl}(2, \mathbb{R})$ doublet:

$$\mathcal{F}_{\mu\nu} = \mathcal{B}_{\mu\nu} - \begin{pmatrix} A_{[\mu, \nu]} \\ \tilde{A}_{[\mu, \nu]} \end{pmatrix}. \quad (35)$$

Using this doublet Townsend generalizes the DBI action to [12]:

$$S_{DBI} = \int d^2\sigma \frac{1}{2v} [\det g + \mathcal{F}^t \mathcal{M} \mathcal{F}]. \quad (36)$$

This action is manifestly $\text{Sl}(2, \mathbb{R})$ invariant. In order to calculate two (m, n) strings interactions, we also need a vertex term added to (36), As usual we take the minimal coupling as:

$$S_{int} = \int d^2\sigma \mathcal{J}_{ab}^t \mathcal{M} \mathcal{F}_{ab}, \quad (37)$$

where \mathcal{J}_{ab} is the current of (m, n) string. In the case of static (m, n) strings:

$$\mathcal{J}_{ab} = \epsilon_{ab} \begin{pmatrix} m \\ n \end{pmatrix}. \quad (38)$$

⁵As explained in [5] the coefficient of $dA^{(4)}$ is $\frac{1}{480}$ for self duality.

So the full action governing the (m, n) dynamics is:

$$S = S_{IIb}^{bulk} + S_{DBI} + S_{int}. \quad (39)$$

The action given above is manifestly $Sl(2, \mathbb{Z})$ invariant.

Before going to brane interactions in detail, let us analyze the action (39). If we solve the equation of motion for A, \tilde{A} we have:

$$\mathcal{F}_{ab} = \epsilon_{ab} \begin{pmatrix} m \\ n \end{pmatrix} + \mathcal{G}_{ab}. \quad (40)$$

where m, n are two constants and \mathcal{G}_{ab} is normal to (m, n) in the moduli space. By quantization arguments the conjugate momentum of A, \tilde{A} (m, n respectively) could only acquire integer values. Inserting (40) into (36) and solving the equation of motion for v :

$$S_{DBI} = \int T_{(m, n)} d^2\sigma \sqrt{\det g} \quad (41)$$

which describes a BPS string with tension:

$$T_{(m, n)}^2 = (m, n) \mathcal{M} \begin{pmatrix} m \\ n \end{pmatrix} = (m + n\chi)^2 + \frac{n^2}{\lambda^2}. \quad (42)$$

The same argument holds for the usual DBI action which describes the arbitrary $(m, 1_p)$ brane ($p > 1$). By taking the constant electric field solutions for A_a , we obtain:

$$S_{DBI} = \int T_{(m, 1_p)} d^{(p+1)}\sigma \sqrt{\det g}, \quad (43)$$

where $T_{(m, 1_p)}$ is given by (18), and $m\lambda$ is the quantum value of NS-NS charge or conjugate momentum of A field living in the D_p -brane. Hence the usual DBI action or the action (36) describes the dynamics of objects with the given mass densities, i.e. D-brane, F-string bound states.

In order to calculate (m, n) strings interactions we use the usual methods [17]. We have to perform the Casimir energy calculations which in the first order (tree diagram) is due to exchange of a single graviton and $\mathcal{B}_{\mu\nu}$ fields:

$$\varepsilon(Y)T = -2k_{10}^2 \int d^{10}x \int d^{10}\tilde{x} [T_{\mu\nu} \Delta^{\mu\nu, \rho\sigma} T_{\rho\sigma} - \mathcal{J}_{ab}^t \mathcal{M} \Delta \mathcal{J}_{ab}], \quad (44)$$

where first term in the brackets shows graviton dilaton contributions, and the second, $\mathcal{B}_{\mu\nu}$ interactions. $\Delta^{\mu\nu,\rho\sigma}$, Δ are the corresponding propagators and $T_{\mu\nu}$, \mathcal{J}_{ab} the related currents:

$$T_{\mu\nu} = \frac{1}{2}T_{(m, n)}\delta(x_\perp) \begin{cases} \eta_{\mu\nu} & \mu, \nu = 0, 1 \\ 0 & otherwise. \end{cases} \quad (45)$$

where x_\perp shows the coordinates normal to string. If we use a gauge in which the dilaton contribution is absorbed in gravity part:

$$\Delta^{\mu\nu,\rho\sigma} = (\eta_{\mu\rho}\eta_{\nu\sigma} - \eta_{\mu\sigma}\eta_{\nu\rho})\Delta. \quad (46)$$

Putting them together, we obtain the Casimir energy:

$$\varepsilon(Y) = -2k_{10}^2 V_{(1)}[\mathcal{J}^t \mathcal{M} \mathcal{J} - T_{(m, n)}^2]G_8(Y^2), \quad (47)$$

where

$$\mathcal{J}^t \mathcal{M} \mathcal{J} = (m + n\chi)^2 + \frac{n^2}{\lambda^2}. \quad (48)$$

When the BPS condition is saturated:

$$\mathcal{J}^t \mathcal{M} \mathcal{J} = T_{(m, n)}^2. \quad (49)$$

And the IR limit of string theory is recovered. So the objects defined by mixed and Chan-Paton boundary conditions are **string theoretic realization of (m, n) string bound states**.

7 Discussion:

We have explicitly constructed the bound states of F-strings and D_p -branes ($(m, 1_p)$ branes) in string theory simply imposing mixed boundary conditions on open strings having their ends on them. Moreover we can construct (m, n_p) bound states by both mixed boundaries and Chan-Paton factors on the ends of open strings. These **bound states** are BPS saturated and their mass formula is given by (18).

Another way of checking their long distance (effective field theory) behaviour, is studying the scattering of an string off these objects which we have not considered here. Through these calculation we can also check their NS-NS charge explicitly.

As we argued these bound states can also be understood by T-duality plus boost in corresponding dual theory.

When we deal with (m, n) strings, we should notice that each of these two (mixed boundary conditions and Chan-Paton factors) do not give an $Sl(2, Z)$ invariant interaction separately, but as we have shown explicitly a combination can give an $Sl(2, Z)$ invariant *result*. Being more precise we are using methods and concepts from open F-string theory which are not $Sl(2, Z)$ invariant, in other words we are doing our calculations in a special point of moduli space ($g_s \ll 1, \chi = 0$). Choosing this special point immediately breaks $Sl(2, Z)$ in calculations but not in the final results.

The symmetry enhancement arguments again holds when we have mixed open strings stretched between branes, this shows a way to make an $Sl(2, Z)$ invariant symmetry enhancement argument for IIB one-branes. Such an argument does not hold for strong coupling limit of IIA theory. In this paper although we mainly discussed one-branes the results seems to hold true for IIB five-branes. For IIB three branes again the symmetry arguments is well matched to $Sl(2, Z)$ because their RR charge is an $Sl(2, Z)$ invariant. Thus in the case of three branes only RR charge determines the gauge group (unlike the (m, n) strings)⁶.

The question of simultaneous presence of two strings with different NS-NS charges (m, m') is not addressed here. Applying the same method we introduced in this paper (mixed boundaries on coordinates parallel to D-brane), the spatial coordinates becomes *non-commutative*. The non-commutativity is controlled by $(m - m')$ factor. On the other hand we know that $(m, 1)$ and $(m', 1)$ strings system do not form a BPS saturated state, this is easily seen from the field theory calculations.

Through field theory calculations, energy of the system discussed above is:

$$\varepsilon = \sqrt{\frac{1}{\lambda^2} + m^2} \sqrt{\frac{1}{\lambda^2} + m'^2} - \left(\frac{1}{\lambda^2} + mm'\right). \quad (50)$$

which vanishes at $m = m'$. This result can be generalized for $(m, n), (m', n')$ strings as:

$$\varepsilon = T_{(m,n)} T_{(m',n')} - T_{(m,n)}^{(m',n')}. \quad (51)$$

where $T_{(m,n)}$ is the BPS mass formula:

$$T_{m,n}^2 = (m, n) \mathcal{M} \begin{pmatrix} m \\ n \end{pmatrix} ; T_{(m,n)}^{(m',n')} = (m' \ n') \mathcal{M} \begin{pmatrix} m \\ n \end{pmatrix}. \quad (52)$$

⁶ We can also extend our method to $(m \ 1_3)$ branes although their NS-NS charge is not $Sl(2, Z)$ invariant since their RR charge is, the symmetry enhancement argument is valid under a $Sl(2, Z)$ transformation.

The interaction vanishes if $\frac{m}{n} = \frac{m'}{n'}$. It seems that non-commutativity is related to the fact that these strings do not form a BPS state.

There are some other solutions in related field theories that have masses proportional to $\frac{1}{g_s^2}$, the NS five branes. So one expects that they can be introduced in string theory too, but till now although some special polarizations of them have been found in M(atrrix)-theories [18], unlike D-branes, there is no string theoretic description of them. We can also think of the bound state of NS five branes with fundamental strings or D-branes. Type IIA NS five branes (or their possible bound states) can be studied via M-theory five branes. Bound state of such branes i.e. H-monopole with F-strings and their T-dual i.e. KK-monopoles with F-strings winding charge has been recently investigated, both in the context of string theory and M-theory [19,20]. In these dyonic states like what we have argued the NS two form charge of a H-dyon is related to momentum modes of compactified F-string. Moreover one can construct bound states of NS-branes with D-branes. If we compactify the M-theory on S^1 on a direction making an angle with one of the internal coordinates of M_5 -brane, we obtain a bound state of D_4 -branes and NS five branes in IIA theory.

NS five branes in IIB theory are quite different; by means of $Sl(2,Z)$ symmetry they are related to D_5 -branes. The symmetry enhancement argument for them is exactly like usual D_5 -branes, i.e. There is an $Sl(2,Z)$ invariant $U(1)$ gauge field living in the bound state of $(1, n)$ five branes and the same holds for NS five brane. Hence we expect the symmetry enhancement in the coincident limit.

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